1. Which of the following statement/s is/are true ?
   I) The set of rational numbers is uncountable.
   II) The set of irrational numbers is countable.
   A) I only B) II only C) both I and II D) none of these

2. The geometric series $1 + x + x^2 + \ldots$ converges if
   A) $-1 \leq x \leq 1$ B) $x < -1$ C) $x \geq 1$ D) $-1 < x < 1$

3. Consider the function, $f(x) = \begin{cases} \frac{|x|}{x}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$
   The left hand limit of f(x) at 0 is
   A) 1 B) -1 C) 0 D) not defined

4. Name the following theorem :
   If a function f defined on [a, b] is continuous in [a, b], derivable in (a, b) and f(a) = f(b), then there exists at least one real number $c \in (a, b)$ such that $f'(c) = 0$.
   A) Rolle's theorem B) Lagrange's mean value theorem
   C) Cauchy's theorem D) Generalized mean value theorem

5. Let f be Riemann integrable over [a, b]. Which of the following statement/s is/are true ?
   I) |f| is Riemann integrable.
   II) f^2 is not Riemann integrable.
   A) I only B) II only C) both I and II D) none of these

6. For an analytic function, which of the following statement/s is/are true ?
   1) The real part of the analytic function is a harmonic function.
   2) The imaginary part is not a harmonic function.
   A) 1 only B) 2 only C) both 1 and 2 D) none of these

7. The value of $\int_C \frac{dz}{z-3}$, where C is the circle $|z - 2| = 5$, is
   A) 2π B) $2\pi i$ C) -2π D) $-2\pi i$

8. The residue of $\frac{ze^z}{(z-1)^3}$ at its pole is
   A) $\frac{3}{2e}$ B) $\frac{2e}{3}$ C) $\frac{3e}{2}$ D) $\frac{3}{2}$

9. If Q, R, C are respectively the fields of rational numbers, real numbers and complex numbers, then which one of the following structures is not a vector space ?

   A) R over the field Q
   B) R over the field R
   C) Q over the field R
   D) C over the field C

10. What is the dimension of the subspace S of R^4 generated by the vectors (2, 1, 1, 0), (1, 0, 2, 1), (3, 2, 0, –1) and (4, 1, 5, 2) ?

   A) 1
   B) 2
   C) 3
   D) 4

11. Let $T : R^2 \rightarrow R^2$ be a linear transformation such that $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\end{bmatrix}$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}2\\1\end{bmatrix}$. What is $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$ equal to ?

   A) $\begin{bmatrix}1\\2\end{bmatrix}$
   B) $\begin{bmatrix}2\\1\end{bmatrix}$
   C) $\begin{bmatrix}3\\3\end{bmatrix}$
   D) $\begin{bmatrix}2\\2\end{bmatrix}$

12. If $A = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$ satisfies the matrix equation $A^2 - kA + 2I = 0$, what is the value of k ?

   A) 0
   B) 1
   C) 2
   D) 3

13. If A is a 2×2 non-singular matrix, then what is adj (adj A) equal to ?

   A) A^2
   B) A
   C) A^{-1}
   D) – A

14. Consider the following statements on trace of matrices :

   1) tr (AB) = tr (BA)
   2) tr (A + B) = tr (A) + tr (B)

   Which of these statement/s is/are correct ?

   A) 1 only
   B) 2 only
   C) both 1 and 2
   D) neither 1 nor 2

15. If A is a square matrix of order 2 such that tr (A) = 7 and | A | = 12, then its eigen values are

   A) 3, 4
   B) 2, 3
   C) 4, 5
   D) 7, 12

16. Matrix of the quadratic form $Q = x_1^2 + x_2^2 - 3x_3^2 + 2x_1x_2 - 6x_1x_3$ is given by

A) $\begin{bmatrix} 1 & 2 & -6 \\ 2 & 1 & 0 \\ -6 & 0 & -3 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ -3 & 1 & -3 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -3 & 0 & -3 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 1 & -3 \\ 1 & 1 & 0 \\ -3 & 0 & -3 \end{bmatrix}$

17. The limit inferior of a sequence $\{A_n\}$ of sets is defined by

A) $\bigcap_{n=1}^{\infty} \bigcup_{k=1}^{\infty} A_k$

B) $\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

C) $\bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

D) $\bigcup_{n=1}^{\infty} \bigcap_{k=1}^{\infty} A_k$

18. A σ-field of subsets of a non-empty set is a class of sets which is closed under
   A) complementation and finite union
   B) union and intersection
   C) complementation and finite intersection
   D) complementation and countable union

19. The Lebesgue outer measure of the interval [0.5, 2.5] is
   A) 0.5 B) 2.5 C) 2 D) 1.5

20. If $\{f_n\}$ is a sequence of measurable functions that is bounded above by an integrable function, which of the following inequalities is true ?

A) $\int \lim_{n\to\infty} \sup f_n d\mu \le \lim_{n\to\infty} \sup \int f_n d\mu$

B) $\int \lim_{n\to\infty} \sup f_n d\mu \ge \lim_{n\to\infty} \sup \int f_n d\mu$

C) $\int \lim_{n\to\infty} \inf f_n d\mu \le \lim_{n\to\infty} \inf \int f_n d\mu$

D) $\int \lim_{n\to\infty} \inf f_n d\mu \ge \lim_{n\to\infty} \inf \int f_n d\mu$

21. Let A and B be two independent events with P(A) = 0.3 and P(B) = 0.4. Then probability that 'B occurs but A does not' is
   A) 0.28 B) 0.18 C) 0.7 D) 0.6

22. Suppose an absent-minded office assistant puts four letters in four addressed envelopes. What is the probability that the assistant will misplace every letter ?
   A) $\frac{5}{8}$ B) $\frac{3}{8}$ C) $\frac{19}{24}$ D) $\frac{5}{24}$

23. A problem is given to three students whose probabilities of solving it independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that none of them solves the problem ?

A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{3}$ D) $\frac{3}{4}$

24. A and B are two events such that P (A $\cup$ B) = 0.8, P(A) = 0.3 and P(B) = p. Then the value of p for which A and B are independent is equal to

A) $\frac{5}{7}$ B) $\frac{2}{7}$ C) $\frac{7}{10}$ D) $\frac{3}{10}$

25. Number of conditions satisfied by 6 events for their mutual independence is

A) 64 B) 63

C) 57 D) 120

26. The set of discontinuity points of a distribution function is

A) finite B) infinite

C) countable D) atmost countable

27. Which of the following statement/s is/are true ?

1) For random variables with finite second moments, one can do better than the Chebychev inequality.
2) If higher moments are assumed to exist, it is not possible to improve Chebychev inequality.

A) 1 only B) 2 only

C) both 1 and 2 D) neither 1 nor 2

28. Which of the following distributions has the characteristic function, $\phi(t) = e^{-|t|}, t \in R$?

A) standard Cauchy B) gamma

C) double exponential D) beta

29. For the characteristic function $\phi(t)$ of a distribution, which of the following is not true ?

A) $\phi(0) = 1$ B) $|\phi(t)| \leq 1, \forall t \in R$

C) $\phi(-t) = \phi^{-}(t)$ D) $\phi(t)$ is not uniformly continuous

30. Which of the following relations among convergence of a sequence of random variables does not hold good ?
    A) almost sure convergence implies convergence in probability
    B) convergence in probability implies convergence in distribution
    C) convergence in probability implies convergence in mean
    D) convergence in r^{th} mean implies convergence in s^{th} mean for $r > s$

31. Let $\{X_n\}$ be a sequence of i.i.d. random variables and for $n \geq 1$, let $S_n = \sum_{k=1}^{n} X_k$. Then $\frac{S_n}{n} \xrightarrow{a.s.} \mu$ if and only if $E|X| < \infty$, where $\mu = E(X)$. This law of large numbers is known as
    A) Kolmogorov's B) Khintchin's C) Chebychev's D) Bernoulli's

32. Which of the following theorems is known as classical central limit theorem ?
    A) De Moivre-Laplace B) Lindberg-Levy
    C) Liapunov D) Lindberg-Feller

33. If $X \sim b(10, 1/2)$, what is the most likely value of X ?
    A) 5 B) 5.5 C) 4 D) 6

34. Which of the following distributions has the property that mean, variance and third central moment are equal ?
    A) Hypergeometric B) Poisson C) Exponential D) Geometric

35. The moment generating function of a distribution is $\frac{1}{2}\left(1-\frac{1}{2}e^{t}\right)^{-1}$. What is the mean of the distribution ?
    A) 1 B) 4 C) 2 D) 3

36. If $X_1, X_2 \ldots, X_n$ are independent exponential variates with mean λ, then min $(X_1, X_2, \ldots, X_n)$ is exponentially distributed with mean
    A) $\frac{n}{\lambda}$ B) $\frac{\lambda}{n}$ C) $n\lambda$ D) $\frac{1}{n\lambda}$

37. If X has the exponential distribution with mean $\frac{1}{\lambda}$ and $Y = 1 - e^{-\lambda X}$, then distribution of the random variable Y is
    A) standard Cauchy B) standard normal
    C) standard exponential D) uniform over (0, 1)

38. If X and Y are i.i.d. lognormal with mean 0 and variance 1, what is the distribution of Z = XY ?
    A) lognormal with mean 0 and variance 2
    B) lognormal with mean 0 and variance 1
    C) normal with mean 0 and variance 2
    D) normal with mean 0 and variance 1

39. Distribution of the ratio of two independent normal variates with common mean 0 and common variance σ^2 is
    A) Laplace
    B) Lognormal
    C) Exponential
    D) Standard Cauchy

40. If $X_1, X_2, \ldots X_9$ are i.i.d. N(0, 1) variates and $U = \frac{1}{3}\sum_{j=1}^{9} X_j$, distribution of the random variable U is
    A) standard normal
    B) normal with mean = 9, variance = 1
    C) standard Cauchy
    D) normal with mean = 3, variance = 1

41. If X_1, X_2, X_3, X_4, X_5 is a random sample from uniform (0, 1) distribution, then distribution of the sample range is
    A) Gamma distribution
    B) Beta distribution of the first kind
    C) Cauchy distribution
    D) Beta distribution of the second kind

42. If (X, Y) has a bivariate normal with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ and ρ, then variance of the conditional distribution of Y given X = x is
    A) $\sigma_1^2\rho^2$
    B) $\sigma_1^2(1-\rho^2)$
    C) $\sigma_2^2\rho^2$
    D) $\sigma_2^2(1-\rho^2)$

43. Let X_1, X_2, X_3, X_4 be four independent standard normal variates. Then distribution of the statistic $\frac{X_1 - X_2}{\sqrt{X_3^2 + X_4^2}}$ is
    A) Student's t with $\sqrt{2}$ d.f.
    B) Normal distribution with mean $\sqrt{2}$ and variance 3
    C) Student's t with 2 d.f.
    D) Normal distribution with mean 2 and variance 3

44. If X_1, X_2, X_3, X_4 is a random sample drawn from the standard exponential distribution, then pdf of the first order statistic $X_{(1)}$, for $x > 0$, is given by

A) $4(1 - e^{-4x})$ B) e^{-4x} C) $4e^{-4x}$ D) $1 - e^{-4x}$

45. If $X_1, X_2, \ldots X_n$ is a random sample of size n drawn from a $N(\mu, \sigma^2)$ – population and $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$, then $V(S^2)$ is given by

A) $\frac{2}{n}\sigma^4$ B) $\frac{2}{(n-1)}\sigma^4$ C) $\frac{1}{n}\sigma^4$ D) $\frac{1}{(n-1)}\sigma^4$

46. The values of the total correlation coefficients and multiple correlation coefficient of a trivariate distribution are given as 0.6, 0.7, 0.8, 0.85. Among these, identify the value of the multiple correlation coefficient.

A) 0.6 B) 0.7 C) 0.8 D) 0.85

47. If correlation coefficient between random variables X and Y is 0.8, then the correlation coefficient between $-2X + 3$ and $Y + 1$ is

A) 0.40 B) -0.8 C) -0.40 D) 0.8

48. The distribution of the square of Student's $t_{(n)}$ variate is

A) $F_{(n, 1)}$ B) Cauchy
C) $F_{(1, n)}$ D) Standard normal

49. Let $X_1, X_2, \ldots X_n$ be a random sample from a distribution with cdf F(.). For a fixed t, the estimator T_n for F(t) defined by $T_n = \frac{1}{n}\{\text{Number of } X_i \leq t\}$ is

A) unbiased and consistent B) unbiased but not consistent
C) consistent but not unbiased D) neither consistent nor unbiased

50. The condition $V(T_n) \to 0$ as $n \to \infty$ for an unbiased estimator T_n to be a consistent estimator is

A) Necessary only B) Sufficient only
C) Necessary and sufficient D) Neither necessary nor sufficient

51. Suppose that U is an unbiased estimator of θ, S is a sufficient statistic and T = E(U/S). Which of the following statements are correct ?

1) T is a function of S, independent of θ
2) T is an unbiased estimator of θ
3) Variance of T is not more than that of U for every θ

A) 1 and 2 only B) 1 and 3 only C) 2 and 3 only D) 1, 2 and 3

52. Let $f_\theta(x) = \frac{1}{\theta} e^{-x/\theta}, 0 < x < \infty$. The estimator of θ obtained by the method of moments is

A) $\bar{X}$ B) $\frac{1}{\bar{X}}$ C) $\frac{1}{\bar{X}^2}$ D) $\bar{X}^2$

53. Let X be a single observation following Poisson distribution with parameter θ. Then the UMVUE of $e^{-\theta}$ is

A) e^{-X}

B) $\delta_1(X) = \begin{cases} 1, & \text{if } X = 0 \\ 0, & \text{if } X \geq 1 \end{cases}$

C) $\delta_2(X) = \begin{cases} 0, & \text{if } X = 0 \\ 1, & \text{if } X \geq 1 \end{cases}$

D) e^X

54. Let $X_1, X_2, \ldots X_n$ be a random sample from uniform over $\left[\theta - \frac{1}{2}, \theta + \frac{1}{2}\right]$. If $L = \min(X_1, X_2, \ldots, X_n)$ and $M = \max(X_1, X_2, \ldots X_n)$ then an mle of θ is

A) $L + \frac{1}{2}$ B) $M - \frac{1}{2}$ C) $\frac{L+M}{2}$ D) $L + M$

55. Cramer-Rao inequality gives

A) a lower bound for the variance of an unbiased estimator

B) a lower bound for the variance of an unbiased estimator under certain regularity conditions

C) a upper bound for the variance of an unbiased estimator

D) a upper bound for the variance of an unbiased estimator under certain regularity conditions

56. If $\hat{\theta}_1$ is a most efficient estimator and $\hat{\theta}_2$ is any other estimator with efficiency e, then the correlation coefficient between $\hat{\theta}_1$ and $\hat{\theta}_2$ is

A) e^2 B) e^{-2} C) $e^{1/2}$ D) $e^{-(1/2)}$

57. If $x_1, x_2, \ldots, x_n$ is a random sample from $N(\mu, 1)$, then an unbiased estimator of $\mu^2 + 1$ is

A) $\frac{1}{n}\Sigma_{i=1}^{n} x_i^2$ B) $\bar{x}^2 - 1$ C) $\bar{x}^2 + 1$ D) $\bar{x}^2$

58. The mean of 16 observations drawn from a normal population with mean μ and standard deviation 4 is 24. What is the 95% Confidence Interval for μ ?

A) (21.04, 26.96) B) (22, 26)

C) (22.04, 25.96) D) (21, 25)

59. Which of the following about the power of a test of H_0 Vs. H_1 is/are true ?

A) Power = probability of rejecting H_0 when H_0 is false

B) Power = 1 – probability of Type II error

C) Power = value of the power function at which H_1 is true

D) All of these

60. If p denotes the P-value of a test and α is the chosen level of significance, then reject H_0 if

A) $p \leq \alpha$ B) $p \geq \alpha$ C) $p \geq \frac{\alpha}{2}$ D) $p \leq \frac{\alpha}{2}$

61. If each observation in a data is replaced by + or – according as its value is above the sample median or below the sample median and the sequence of such markings are as given below :

+ – – + + + – + – – – – + – + – – + then the value of the test statistic for testing H_0 : "The given sample is a random sample" is equal to

A) 18 B) 11 C) 10 D) 8

62. Which of the following is/are assumptions for Student's t-test for difference of means ?

A) The parent populations from which the samples drawn are normal

B) The two random samples are independent of each other

C) The population variances are equal and unknown

D) All of these

63. For SPRT, which of the following statement/s is/are true ?

I. The sample size n is a random variable.

II. The SPRT terminates with probability 1.

A) I only B) II only C) Both I and II D) None of these

64. Which of the following statistics is/are known as Kolmogorov-Smirnouv statistics ?

A) $D_n^+ = \sup_x [F_n^*(x) - F(x)]$ B) $D_n^- = \sup_x [F(x) - F_n^*(x)]$

C) $D_n = \max(D_n^+, D_n^-)$ D) All of these

65. If a sample of size 3 is drawn from a population of size 5 by the method of SRSWOR, then the probability of selecting a specified unit of the population into the sample would be

A) $\frac{3}{5}$ B) $\frac{1}{5}$ C) $\frac{1}{10}$ D) $\frac{1}{125}$

66. A simple random sample of 6 households was drawn from a village containing 100 households. The number of persons per household in the sample were 5, 6, 4, 7, 3 and 2. The estimate of the total number of people in the village is

A) 27 B) 625 C) 450 D) 3125

67. Using SRSWOR, a sample of size 16 taken from a population of 50 units provides $s^2 = 4$, then estimated S.E. of the estimate of the population mean is

A) $\frac{17}{100}$ B) $\frac{\sqrt{17}}{10}$ C) $\frac{6}{25}$ D) $\frac{17}{50}$

68. A population of size 100 is divided into 3 strata with sizes 50, 30 and 20 respectively with equal variance. What are the stratum sample sizes under proportional allocation for a fixed sample size 20 ?

A) 4, 9 and 7 B) 6, 9 and 5 C) 10, 6 and 4 D) 8, 7 and 5

69. If a stratified sample of size 45 is to be selected by Neyman allocation from a population with $N_1 = 150$, $N_2 = 350$, $S_1^2 = 4$, $S_2^2 = 9$, then the number of units to be selected from the first stratum is

A) 20 B) 35 C) 75 D) 10

70. A linear systematic sample is drawn from a population of 40 units with a sampling interval of 7. An estimate of the population total is

A) 40 × sample mean B) 7 × sample mean

C) sample total D) 7 × sample total

71. Given that N = 500, n = 50, $\bar{x} = 125$, $\bar{y} = 250$, $\bar{X} = 150$. Then the ratio estimator of $\bar{Y}$ is

A) 325 B) 375 C) 300 D) 400

72. A systematic sample of size n drawn from a population of kn units is considered as a simple random sample of one cluster unit drawn from a population of

A) k cluster units B) n cluster units

C) (nk)/2 cluster units D) (k + n) cluster units

73. Which one of the following refers to the deliberate introduction of non-orthogonality in a design ?

A) local control
B) randomization
C) confounding
D) error control

74. Which of the following forms is/are linear contrasts ?

I) $y_1 - 2y_2 + 2y_3$
II) $3y_1 + 4y_2 - 7y_3$

A) I only
B) II only
C) both I and II
D) None

75. Which of the following principles of experimentation is/are used in the case of CRD ?

A) local control
B) randomization and local control
C) replication and local control
D) replication and randomization

76. For a 2^3-factorial design with r replications, what is the error df ?

A) $6(r - 1)$
B) $7(r - 1)$
C) $7r$
D) $8r$

77. In a RBD with r blocks, t treatments and one missing observation, the error degrees of freedom is

A) $rt - r - t$
B) $rt - r$
C) $rt - r - 1$
D) $rt - r - t + 1$

78. While analysing the data of m × m LSD, the error degrees of freedom in analysis of variance is equal to

A) $(m - 1)(m - 2)$
B) $m(m - 1)$
C) $(m^2 - m - 1)$
D) $(m^2 - m - 2)$

79. For a BIBD with parameters $v = b = 7$, $r = k = 4$, $\lambda = 2$, the number of treatments common between any two blocks is

A) 1
B) 2
C) 3
D) 4

80. In a 2^5 Factorial experiment with factors A, B, C, D, E the interactions ABCD and CDE are confounded. Then which one of the following interactions get confounded automatically ?

A) AB
B) CD
C) DE
D) ABE

Space for rough work

Space for rough work

Space for rough work

Space for rough work